Combination of results from different measurements - an everlasting problem -

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This is a technical paper

– not a political paper !

The change of title now reading

Combination of results from several measurements, Metrological or management effort

has not been authorised

Where the problem is

Given: measurement results $a \pm u(a)$, $b \pm u(b)$

Wanted: (1) c = mean (a, b)

(2) $u(c)$, considering $u(a)$, $u(b)$

What can go wrong with mean values (1)

Default choice: $c = \frac{a+b}{2}$

Refinement: $c_w = \frac{w_a \cdot a + w_b \cdot b}{w_a + w_b}$

***More than 2 values*:** **Outliers !!!**

Departure from normal

What can go wrong with mean values (2)

Outliers & departures from normal …

… robust means , e.g. the median

? **? ? ?** **? ? ?** **? ?**

Fine, but *u*(median) = ???

More about this topic from Dave Duewer in Session 3

What can go wrong with the uncertainty of mean values (1)

Estimate from MU propagation for *c* = ½ (*a* + *b*)

$$u(c) = \frac{1}{2}\sqrt{u^2(a) + u^2(b)}$$

Problem 1: Discrepant results

100 ± 3 ; 140 ± 4 combined into 120 ± 2.5 ???

What can go wrong with the uncertainty of mean values (2)

Problem 1 cont´d: How to assess consistency

$$(b-a) \le u(b-a) \qquad (b-a)^2 \le u^2(b-a) = u^2(a) + u^2(b)$$

How to address lack of consistency

Introduce an extra uncertainty $u^2(a) \rightarrow u^2_{am}(a) = u^2(a) + u^2_{ex}$
(same for *b*)

such that $(b-a)^2 = u^2(a) + u^2(b) + 2u^2_{ex}$

What can go wrong with the uncertainty of mean values (3)

Problem 1 cont´d: How to address inconsistency

- **Solve for the extra uncertainty**
- **Calculate amended uncertainties for *a* and *b***
- **Calculate amended uncertainty of mean value *c***

$$u(c) = \frac{1}{2}\sqrt{u^2(a) + u^2(b) + 2u_{ex}^2}$$

Simple approach considered here: $u(c) = \frac{1}{2}(b - a)$

Other approaches: **Some combination of** $u(a)$, $u(b)$, $(b - a)$

What can go wrong with the uncertainty of mean values (4)

Problem 1 cont´d: How to address inconsistency

Techniques from classical statistics carry over to MU

- **Analysis of variance (ANOVA)**
- **Chi-squared techniques**
- **Maximum likelihood techniques**

New approaches derived from GUM principles

- **Bias correction & correction uncertainty**

What can go wrong with the uncertainty of mean values (5)

Problem 2: Correlated results

Measurements with common error are correlated!

$$u^2(x) = u^2_{rand}(x) + u^2_{syst}(x)$$

$$u^2(\bar{x}) = \frac{u^2_{rand}(x)}{N} + u^2_{syst}(x)$$

What can go wrong with the uncertainty of mean values (6)

Problem 2 cont´d: How to address correlation

In absence of relevant information

$$\frac{u_{single}}{\sqrt{N}} \leq u_{mean} \leq u_{single}$$

- **Factor 1 / SQR(*N*) makes a big difference for N = 10**
- **Claim requires demonstration of independence**

Final remarks (1)

Complementary practical relevance

Origin of results	Discrepancy	Correlation
different replicates	low	high
different runs	moderate	high
different laboratories	moderate	moderate
different methods	high	low

Final remarks (2)

- **There is no master approach giving the definitive solution – results depend on the method / model**
- **Comprehesive paper for the proceedings – preprint available at request**

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